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TURBULENT HEAT TRANSFER TO DILUTE AQUEOUS SUSPENSIONS OF TITANIUM DIOXIDE IN PIPES

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Abstract—An experimental investigation into turbulent heat transfer to pseudoplastic titanium dioxide suspensions in pipes has been carried out. Existing heat transfer correlations, including the analogy equations between heat and momentum transfer, generally predict higher Nusselt and Stanton numbers than those observed experimentally. However, a simplified heat transfer model based on the consideration of the laminar sub-layer at the wall and the turbulent core correlates the heat transfer results for heating as well as cooling. The limitations of the existing analogy equations between heat and momentum transfer are discussed.

INTRODUCTION

Although turbulent flow is desirable because of increased rates of heat transfer, the attainment of turbulence is often difficult because of the high consistency of many non-Newtonian fluids. However, there are some slurries which are highly pseudoplastic and of low consistency, so that turbulent conditions can be achieved. The complicated nature of turbulent flow prevents an analytical solution for heat transfer and, unlike laminar flow, the analysis of the turbulent heat transfer data involves dimensional analysis which results in empirical correlations. An alternative approach is based on the analogy between the processes of momentum and heat transfer.

REVIEW OF PREVIOUS WORK

Empirical correlations

Several workers, notably Winding *et al.* (1944) and Orr & Dalla Valle (1954), have investigated the turbulent heat transfer characteristics of suspensions which are slightly pseudoplastic. Orr and Dalle Valle correlated their data on chalk slurries with a Dittus–Boelter type equation. They also defined the fluid properties such as viscosity, heat capacity and thermal conductivity to be used in the evaluation of the various dimensionless groups in the correlation.

Thomas (1960) presented some turbulent heat transfer data on aqueous thoria suspensions in the form of a heat transfer *j*-factor analogous to the Colburn *j*-factor. The slurries were found to behave as Bingham plastics. Thomas used the Hedström criterion to predict the transition from laminar to turbulent flow.

The relevant dimensionless groups normally used in the case of Newtonian turbulent heat transfer were generalised for application to non-Newtonian fluids by Metzner *et al.* (1957).

This involved the replacement of Newtonian viscosity with a suitable apparent viscosity, μ_{apparent}, obtained by equating the generalised Reynolds number, Re′, to the Newtonian Reynolds number, Re, giving

$$\mu_{\text{apparent}} = \gamma(\bar{u}/D)^{n'-1}, \text{ where } \gamma = 8^{n'-1} K'$$

where n' and K' are the generalised flow behaviour index and consistency index respectively, which are obtained from pipe-flow experiments and defined by Metzner & Reed (1955) in terms of wall shear stress, τ_w, mean velocity, $\bar{u}$, and tube diameter, D, as $\tau_w = K'(8\bar{u}/D)^{n'}$.

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The generalised Prandtl number, Pr′, likewise becomes

$$\mathrm{Pr}' = (C_p\gamma/k)\,(\bar{u}/D)^{n'-1}$$

where C_p and k are heat capacity and thermal conductivity respectively.

The Sieder–Tate viscosity correction factor is replaced by $(\gamma_b/\gamma_w)^{0.14}$, where the subscripts b and w mean γ being evaluated at average bulk and average wall temperative respectively.

The final correlation involving Nusselt number, Nu, then becomes

$$\mathrm{Nu} = 0.023\ \mathrm{Re}'^{0.80}\ \mathrm{Pr}'^{0.40}\ (\gamma_b/\gamma_w)^{0.14}. \tag{1}$$

Some limited data on heat transfer to power law fluids in turbulent flow through circular pipes were correlated by Clapp (1961) with two relations of unusual type.

Tokura & Yoshifumi (1967) correlated their experimental data on turbulent heat transfer to pseudoplastic and dilatant fluids in circular and rectangular ducts by the following relation

$$\mathrm{Nu} = 0.023^{5\beta}\,[(\bar{u}D\rho/K)\,(\bar{u}/D)^{n-1}]^{4\beta}\,[(c_pK/k)\,(\bar{u}/D)^{n-1}]^{1/3} \tag{2}$$

where $\beta = (n+2)/(4n+1)3$; n and K being parameters of power law model, $\tau = K(-\mathrm{d}u/\mathrm{d}r)^n$, where τ and $(-\mathrm{d}u/\mathrm{d}r)$ are shear stresses and shear rate respectively, and ρ the density. They also presented a correlation for Bingham plastics.

Analogy between momentum and heat transfer in non-Newtonian fluids

The modified momentum and heat transfer analogy of Reichardt (1957) was extended to fluids of high Prandtl number by Metzner & Friend (1958, 1959), and for Newtonian fluids the result is

$$\mathrm{St} = \frac{\frac{1}{2}f\theta}{\phi + 11.8\sqrt{(f/2)}\,(\mathrm{Pr}-1)\,\mathrm{Pr}^{-1/3}}. \tag{3}$$

In [3] St, Pr and f designate Stanton number, Prandtl number and Fanning friction factor respectively; and constant values 1.0 and 1.2 were proposed for θ (ratio of maximum to mean temperature difference) and ϕ (ratio of maximum to mean velocity) with the limitation, f Pr $\mathrm{Re}^2 > 5\times10^5$.

For non-Newtonian systems the thermal boundary layer will be confined to the wall region, and in this region the shear stress, and hence the apparent viscosity, will be nearly constant. There is therefore little difference between Newtonian and non-Newtonian systems. Using this argument, Metzner & Friend (1959) suggested that [3] should also be applicable to non-Newtonian fluids provided the Prandtl group is evaluated using the apparent viscosity corresponding to wall shear stress and the friction factor, f, is calculated from the logarithmic resistance formula of the von Karman type as presented by Dodge & Metzner (1959).

Clapp (1961) extended Martinelli's analogy between momentum and heat transfer to power law fluids. Nusselt numbers predicted by the analogy were substantially below the measured values. Experimentally observed temperature distributions qualitatively show that the major portion of temperature change occurs in a region very close to the wall.

Tokura & Yoshifumi (1967) found that the profiles of eddy diffusivities for momentum and heat transfer with non-Newtonian fluids were just the same as those for Newtonian fluids if the Reynolds numbers defined in [2] are the same.

Petersen & Christiansen (1966) have suggested a minor improvement of [3] by defining an effective Prandtl number for deviation from Newtonian behaviour and, in addition, correlations were proposed for θ and ϕ in [3]. Slightly better correlation with Friend's data was claimed.

Hughmark (1969) has considered momentum, heat and mass transfer analogies for turbulent flow pipes. This method, which depends on the numerical integration of the equations for the turbulent velocity profile, is said to be applicable to non-Newtonian fluids because it is claimed that the local apparent viscosity can be related to the local shearing stress. The additional complications of Petersen and Christiansen and those of Hughmark are not justified since the improvement over [3] is not significant. Hughmark's approach has also been criticised by Thomas (1970).

Harris & Wilkinson (1971) have presented another extension of the Taylor–Prandtl analogy. In this the rheological properties of the fluids are characterised by five parameters, three of which are applicable in the turbulent core and which are given by Bowen's formula, [4], and the remaining two are applicable in laminar sub-layer, given by a laminar rheological model, the power-law model in this case. Bowen's formula is written as

$$\tau_w D^x = A\,\bar{u}^w \qquad [4]$$

where τ_w is the wall shear stress, $\bar{u}$ the mean velocity, D the pipe diameter and A, x and w are experimentally derived parameters. The final correlation takes the form:

$$\mathrm{St} = \frac{\frac{1}{2}f}{1 + a\Phi^b N_{\mathrm{Pr}}^c\,(N_{\mathrm{Pr}} - 1)} \qquad [5]$$

where a, b and c are functions of the 5-rheological parameters x, w, A, n and K. Φ and N_{Pr} are non-Newtonian forms of the Reynolds and Prandtl numbers respectively.

EXPERIMENTAL RESULTS

The aqueous suspensions of titanium dioxide studied were pseudoplastic and the laminar flow characteristics are represented by the relation of Metzner & Reed (1955), viz.

$$\tau_w = K'(8\bar{u}/D)^{n'}.$$

The suspensions studied are listed in table 1 with the approximate values of the index n' and the constants x and w of Bowen's formula.

Tiona WD is the commercial name of a titanium dioxide manufactured by Laporte Industries Ltd., England. A small amount of an organic acid was added to suspensions to improve dispersion properties.

The suspensions were characterised in a capillary tube viscometer using one tube diameter at different temperatures to cover the temperature range encountered during the heat transfer experiments. A typical logarithmic plot of τ_w vs $(8\bar{u}/D)$ for a 15 per cent titanium dioxide suspension is shown in figure 1. The large extrapolation of these curves is possible without any serious error. This is well demonstrated on f–Re′ plots of viscometric and isothermal pressure

Table 1. Rheological data for the suspensions studied

Concentration % wt.	Temperature °C.	n'	x (dimensionless)	w
5% Tiona WD	11.00-54.4	0.88	0.0	1.79
10% Tiona WD	10.25-59.8	0.847	0.0	1.78
15% Tiona WD	16.80-55.7	0.592	0.0	1.75
25% Tiona WD	20.70-56.9	0.422	0.0	1.70
Water	8.50-59.4	1	0.255 - 0.185	1.75

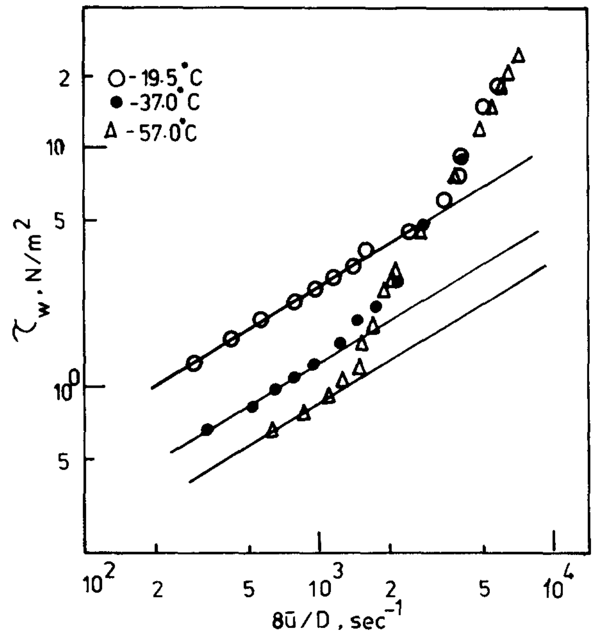

Figure 1. $\tau_w - 8\bar{u}/D$ plot for viscometric data, 15 per cent TiO_2.

drop data of these fluids, figure 2, for a 15 per cent titanium dioxide suspension. The densities and specific heats of the suspensions were taken as the weighted average of the values of individual constituents and the thermal conductivity was calculated from the formula proposed by Orr & Dalla Valle (1954).

Heat transfer and isothermal pressure drop measurements were carried out on two tubes of i.d. 1.27 and 1.91 cm. Non-isothermal pressure drop data were taken simultaneously with heat transfer measurements. Heating measurements were carried out in both tubes but cooling experiments were carried out only in the 1.27-cm tube. For heating, steam was used for the 1.91-cm tube and hot water for the 1.27-cm tube. (Variations of tube wall temperature along the

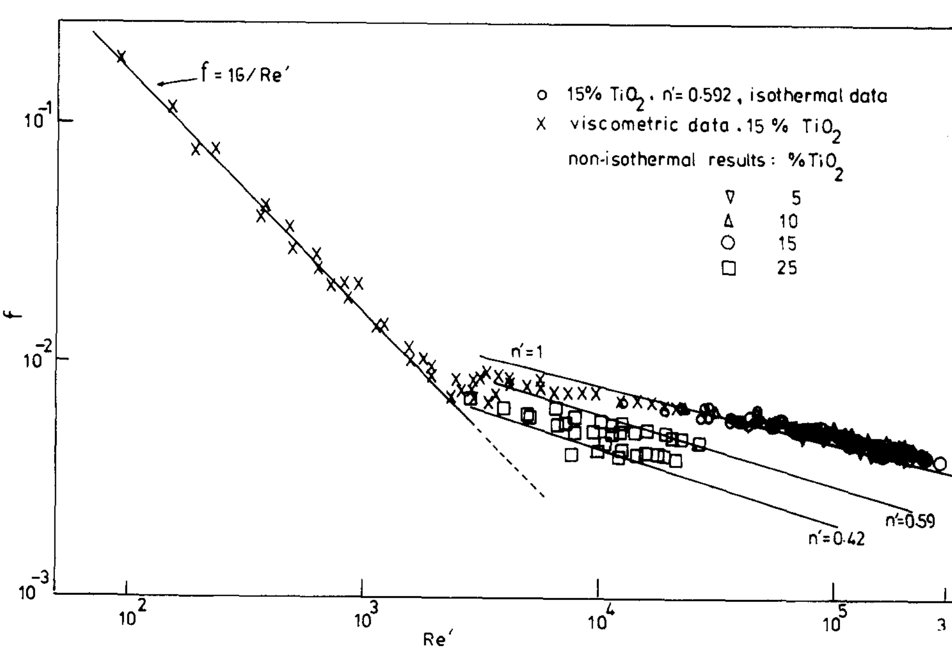

Figure 2. f – Re′ plot.

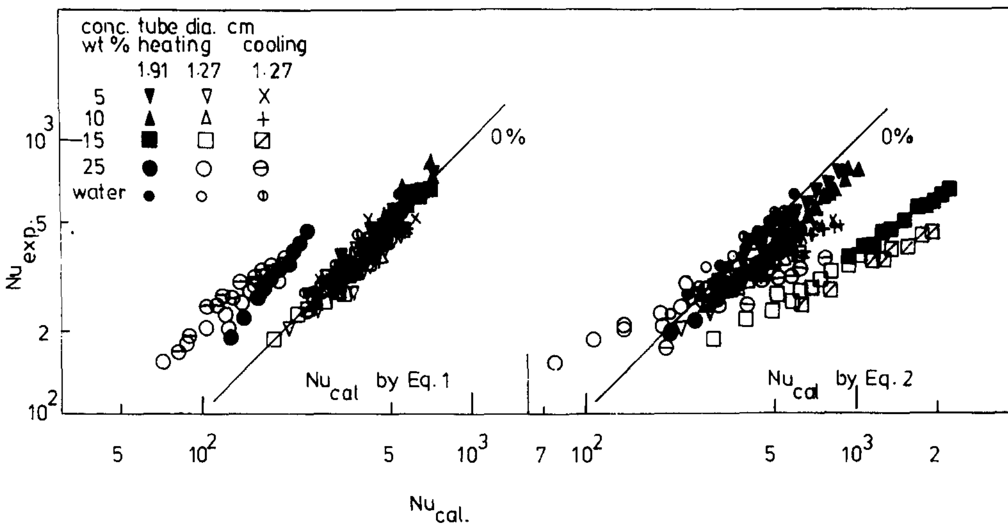

Figure 3. $Nu_{exp} - Nu_{cal}$ plot.

length of the tubes were very small.) The details of the experimental equipment and procedure can be found elsewhere in Quader (1972).

HEAT TRANSFER RESULTS

Experimental results and their comparison with the predicted values from available correlations

In figure 3 are shown the plots of expermientally measured Nusselt number, Nu_{exp} against calculated Nusselt number, Nu_{cal}, from the available correlations given by [1] and [2]. Equation [1] was used without the viscosity correction factor. For cooling runs only [1] was used with 0.30 for the exponent of the Prandtl group.

In figure 4 are shown the plots of experimentally observed Stanton number, St_{exp} against calculated Stanton number, St_{cal} from [3] given by Metzner & Friend (1959).

While evaluating the various groups of the analogy equation of Harris & Wilkinson (1971), [5], the group Φ is unity when x is zero. Equation [5] was rearranged by making $\Phi = 1$ in the following form

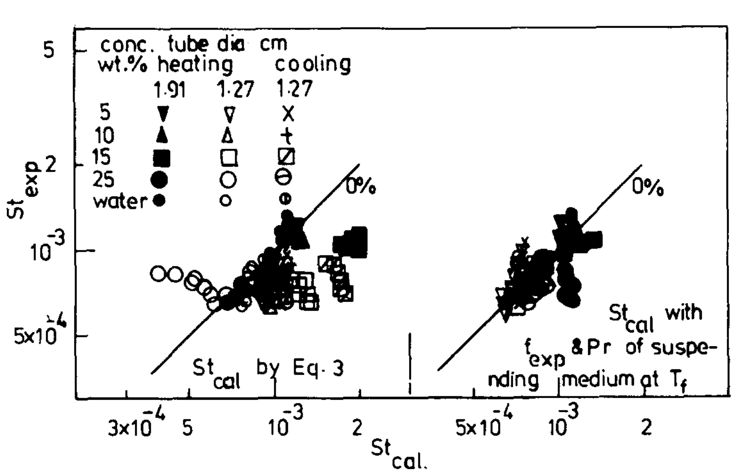

Figure 4. $St_{exp} - St_{cal}$ plot.

form

$$[(\tfrac{1}{2}f/\mathrm{St})-1]/(N_{\mathrm{Pr}}-1)=aN_{\mathrm{Pr}}^{c} \qquad [5a]$$

N_{Pr} in [5] or [5a] is, in fact, the Prandtl group evaluated at the wall shear stress. However, the plot of $[(\tfrac{1}{2}f/\mathrm{St})-1]/(N_{\mathrm{Pr}}-1)$ vs N_{Pr} showed poor agreement with the experimental data.

The conventional f-Re plots for heat transfer runs are shown in figure 2, along with theoretical lines for various values of n' as calculated from the Dodge–Metzner relation [3]. Re' is evaluated at the average bulk temperature of the suspension.

New correlations for the heat transfer results

(a) *correlations involving Re', Pr'*, n' *and Nu.*. After the generalisation of the dimensionless groups involved in non-Newtonian turbulent heat transfer, the form of the relationship obtained for fully developed turbulent flow for a power law fluid is

$$\mathrm{Nu}=F(\mathrm{Re}',\mathrm{Pr}',n')\,. \qquad [6]$$

On this basis the following correlation was found to fit both heating and cooling data with an S.D. of ± 15 per cent.

$$\mathrm{Nu}=0.0561\,(\mathrm{Re}')^{0.697}\,(\mathrm{Pr}')^{0.549}. \qquad [7]$$

All the relevant properties were evaluated at the average bulk temperature of the fluids. In figure 6, $\mathrm{Nu}_{\mathrm{exp}}$ vs $\mathrm{Nu}_{\mathrm{cal}}$ from [7] are plotted and the agreement is seen to be reasonable.

(b) *Application of Bowen's formula to turbulent heat transfer.* In order to explore the application of Bowen's formula to turbulent heat transfer in smooth tubes, the heat transfer coefficient, h, was considered to be some function of Bowen's constants x, w, A and the other usual parameters such as $\bar{u}$, D, ρ, C_p, K, L. By dimensional analysis, we then get

$$\mathrm{St\ or\ Nu}=F\left(\frac{\bar{u}^{2-w}D^{x}\rho}{A},\frac{C_pA}{k}\left(\frac{D^{1-x}}{\bar{u}^{1-w}}\right),x,w,\frac{L}{D}\right). \qquad [8]$$

The groups $(\bar{u}^{2-w}D^{x}\rho/A)$ and $(C_pA/k)(D^{1-x}/\bar{u}^{1-w})$ are defined as new forms of the Reynolds and Prandtl groups respectively containing the constants of Bowen's formula and are denoted as Re_B and Pr_B respectively. L/D is the ratio of length to diameter.

In the above discussion, the application of dimensional analysis is confined to the turbulent core. A more realistic analysis requires the inclusion of the laminar sub-layer and the buffer zone with appropriate rheological models. In the absence of a rheological model for the buffer zone, the heat transfer coefficient, h, is considered to be some function of Bowen's constants x, w, A; the laminar flow characteristics n and K for power law fluids, and the usual parameters $\bar{u}$, D, ρ, C_p, k, L. By dimensional analysis we then obtain

$$\mathrm{St\ or\ Nu}=F\left(\mathrm{Re}_B,\,\mathrm{Pr}_B,\,\frac{C_pK}{k}\left(\frac{\bar{u}}{D}\right)^{n-1},\,x,\,w,\,n,\,\frac{L}{D}\right). \qquad [9]$$

In the light of the findings that (i) x is zero for many fluids as seen from table 1, (ii) x, w do not have any effects on f-Re_B diagram as seen from figure 5, for non-isothermal and isothermal data, (iii) x, w, A, n', K' do not have any relation to each other, then for the case of fully developed flow, the above relation may be rewritten by replacing the Prandtl group $[C_pK/k(\bar{u}/D)^{n-1}]$

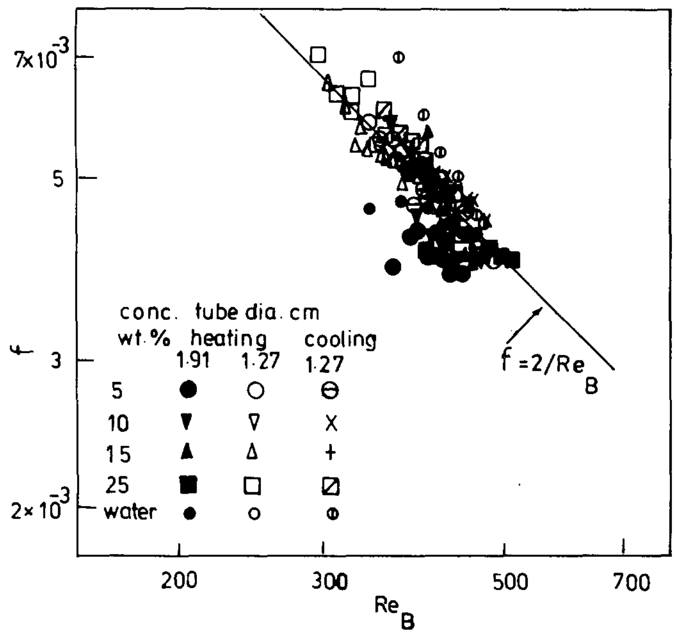

Figure 5. $f-\mathrm{Re}_B$ plot for non-isothermal data.

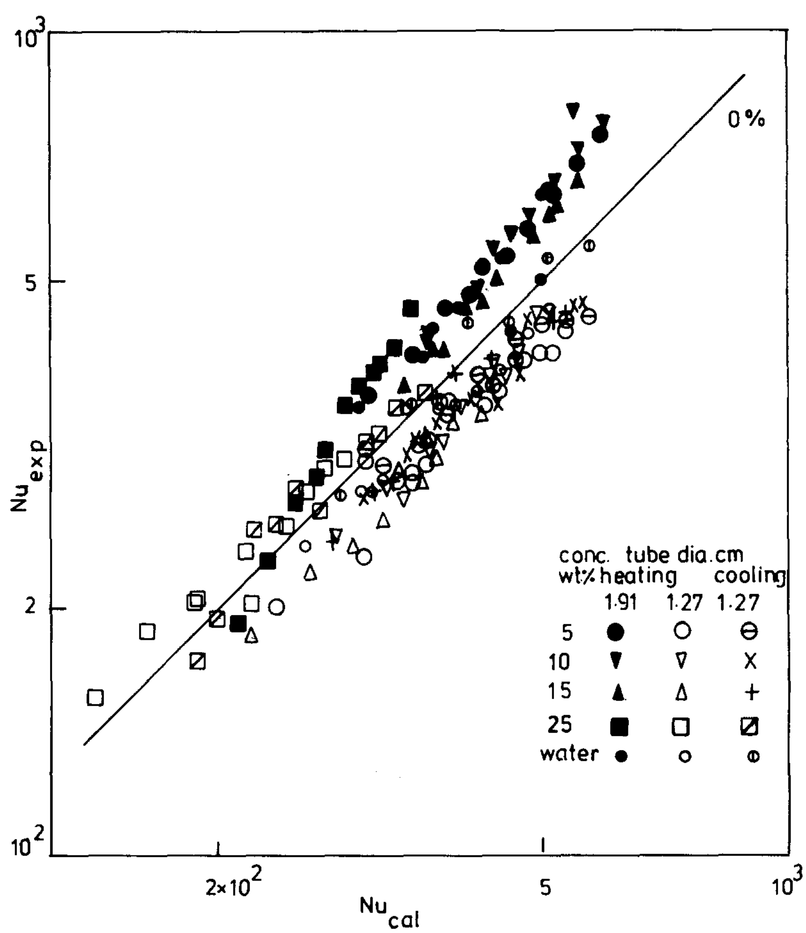

Figure 6. $\mathrm{Nu}_{exp}-\mathrm{Nu}_{cal}$ plot, Nu_{cal} by [7].

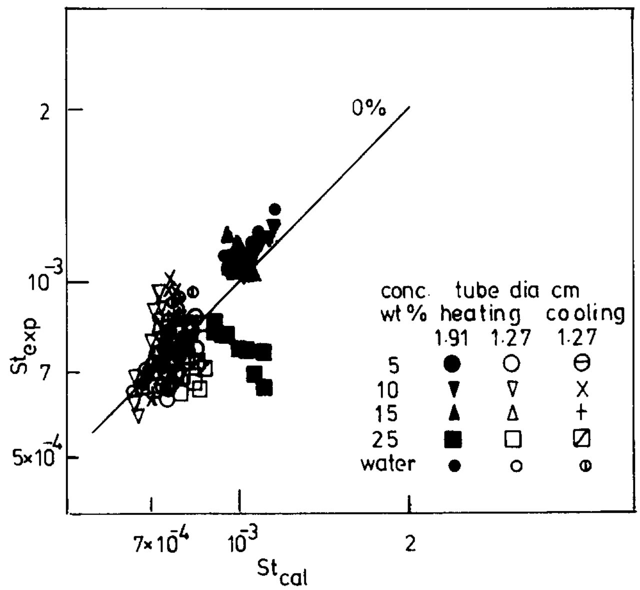

Figure 7. $St_{exp} - St_{cal}$ plot, St_{cal} by [12].

with the generalised Prandtl group $[C_p\gamma/k(\bar{u}/D)^{n-1}]$ to give

to give

$$\text{St or Nu} = F\,(\text{Re}_B, \text{Pr}_B, \text{Pr}'). \qquad [10]$$

However, heat transfer data could not be correlated with this form of correlation without large deviations.

(c) *Turbulent heat transfer model for suspensions.* In a suspension the solid particles are surrounded by the suspending medium and at the wall the solid particles will be separated from the wall by a thin layer of suspending medium. In turbulent flow, this layer at the wall will constitute the laminar sub-layer where resistance to heat transfer is mostly encountered.

In the absence of a suitable rheological model for the buffer zone, the heat transfer rate in turbulent flow is considered to be controlled by (i) the laminar sub-layer at the wall, and (ii) the turbulent core which makes up the total volumetric flow for all practical purposes.

In the laminar sub-layer the heat transfer is considered to be some function of Prandtl group, Pr_w, based on thermal and rheological properties of the suspending medium at the wall shear stress and the average film temperature, T_f. In the present case, Pr_w is taken to be the same as water.

In the turbulent core, the heat transfer is considered to be some function of the rate of momentum transport expressed in terms of the friction factor, f, or preferably by Re_B, which is explicitly related to f. Re_B is to be evaluated at the average bulk temperature.

In view of the usual momentum and heat transfer analogy equation, the following form of relationship was sought,

$$\text{St} = F\,(\text{Re}_B, \text{Pr}_w) \qquad [11]$$

and

$$f = 2/\mathrm{Re}_B.$$

The best correlation obtained in the above form is

$$\mathrm{St} = 0.0989\ (\mathrm{Re}_B)^{-0.697}\ (\mathrm{Pr}_w)^{-0.413} \qquad [12]$$

with an S.D. of ± 13.6 per cent. In figure 7 St_{exp} vs St_{cal} from [12] is plotted.

DISCUSSION

From the plots Nu_{exp}–Nu_{cal} using the correlations given in [1] and [2], figure 3, it is evident that the available correlations predict higher Nusselt numbers in most cases except for some runs with 25 per cent suspensions. The correlations proposed by Clapp (1961) also failed to correlate the experimental results.

The momentum and heat transfer analogy equation of Friend & Metzner (1959) and [3] predicts higher Stanton numbers than those obtained experimentally except for some runs with 25 per cent suspensions as seen from figure 4. The prediction of higher St with [3] is to some extent due to (i) f is calculated from the Dodge and Metzner relation which has already been found to be unsatisfactory for the fluids under consideration and figure 2 shows $f_{exp} > f_{cal}$, and (ii) Pr is evaluated at the average bulk temperature and used the same f-Re′ relation.

Use of the experimental friction factor and evaluation of Pr_w with the properties of the suspending medium at the average film temperature T_f, improve the correlation [3], considerably as seen from figure 4.

The application of Bowen's formula to turbulent heat transfer without the considerations of the laminar sub-layer and the buffer zone is an over-simplification of the heat transfer process. Therefore, such an analysis is unlikely to produce any useful correlations. Even when the laminar flow properties of the fluids at the wall were included in the analysis, the heat transfer results could not be correlated satisfactorily by [10], because the increases in Re_B, Pr_b and Pr′ are out of proportion with increases in Nu or St. In general, the groups Re_B and Pr_B have similar values at the same velocity for the fluids so far studied but Pr varies greatly from fluid to fluid.

The empirical correlation involving Nu, Re′ and Pr′ given by [7] and the simplified heat transfer model in terms of the laminar sub-layer of the suspending medium and turbulent core, given by [12] correlate the heat transfer data well except for data on 25 per cent suspensions for which $n' = 0.42$. For suspensions having $n' < 0.60$, [7] may predict lower heat transfer rates whereas [12] may predict higher heat transfer rates than those actually observed.

The correlation given in [7] requires only the laminar flow properties K' and n', whereas [12] requires the knowledge of x, w and A in the turbulent core as well as the laminar flow characteristic of the suspending medium at wall shear stress. However, [12] is considered to be useful in view of its simple form. Without actual measurements of the velocity and temperature profiles in turbulent flow, improvements on the analogy equations between momentum and heat transfer are not worth attempting.

The range of variables for which [7] and [12] are valid are

$$3.0 \times 10^3 < \mathrm{Re}' < 2.77 \times 10^5$$
$$2.0 < \mathrm{Pr}' < 64$$
$$1.0 > n' > 0.422$$

with Pr_w the same as for water.

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